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Studying the statistical properties of particle counting with a very simple device

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Abstract
A very simple procedure for studying the statistical properties of particle counting is shown. The output of a Geiger tube is connected to the parallel port of a PC which writes onto a file the time at which a particle was detected. From these data both the distribution probability $P(n)$ of counting n particles in a given time and the probability distribution $P(t)$ of time intervals can be easily measured. The device also allows the implementation of an artificial dead time. Effects of dead time on the above distributions are studied. Further exercises are suggested.

This article is dedicated to the memory of Professor Jose Campos Gutierrez who introduced the authors to the 'mysteries' of counting statistics.

1. Introduction

Many situations in everyday life consist of a sequence of random events: arrivals at a serving window, births and deaths, and telephone calls to name but a few. In science many processes are also random such as nuclear disintegration, atomic and molecular de-excitation and electron emission. Radioactive samples provide a handy source of random events in the student laboratory. Because of the statistical nature of nuclear disintegration, the number of ionizing particles emitted by a radioactive source in a given time is subject to fluctuations which follow the Poisson statistics [1–3]. In fact the conditions for the Poisson statistics are also fulfilled in many other counting experiments, for instance, in high-energy physics [4]. Understanding counting statistics is of utmost importance since the associated fluctuations are very often the main source of experimental uncertainties.

Many properties of counting statistics can be easily studied in the student laboratory by representing on a histogram the number of particles emitted by a large-period radioactive source in a given time interval. Particle detection can be achieved using a Geiger tube or

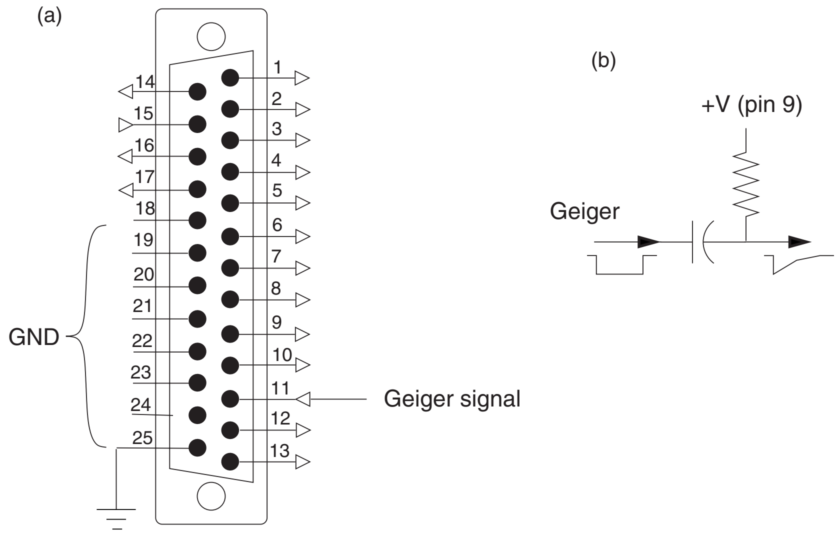

Figure 1. (a) Parallel port connections. (b) RC circuit for shortening a long Geiger signal.

a scintillator detector. Although, in principle counting could be carried out by hand using a simple counter an automatic procedure is more convenient to get enough statistics in a reasonable time. For this end, many commercial data-taking cards are available for PCs.

As a consequence of Poisson statistics, the probability distribution of time intervals, that is, the probability for a random event to be followed by another one in time t, decays as an exponential function of t. Timing detector signals are usually carried out by means of a time-to-amplitude converter (TAC) which gives an output of amplitude proportional to the time interval between the stop signal and the start signal. The amplitude spectrum which can be registered by a multichannel analyser, provides the time interval distribution.

Unfortunately, professional nuclear electronics for the counting and timing of the output signals of a nuclear detector is expensive. Several methods suitable for the student laboratory have been proposed [5, 6]. In this paper a very simple procedure is shown where a PC writes on a file the time at which a Geiger tube is fired without need for any additional electronic unit/card. Using this procedure, properties of the Poisson statistics (section 3), as well as the effects of dead time on particle counting (section 4), can be easily shown in the student laboratory.

2. Counting and timing Geiger signals

As already mentioned, counting and timing of the signals from a nuclear detector are usually achieved by means of electronics for pulse signal processing [3, 4]. However, output signals at a low rate can also be processed by using the parallel port of a PC with no additional electronics. This technique, which will be described next, has been applied in this work for the counting and timing of Geiger signals.

Although the PC parallel port was originally designed to attach printers, it can be used as a general port for any device or application that matches its input/output capabilities. In its standard configuration, it is provided with 12 TTL-buffer output points (figure 1(a)), which are latched and can be written and read under program control using the processor *In* or *Out* instruction. This port also has five steady-state input points that may be read using the processor's *In* instruction. In addition, one input can also be used to create a processor interruption under program control. The input/output signals are made available at the 25-pin, D-type female connector. Assuming the output of the Geiger tube is a 5 V pulse, the experiment requires no hardware besides just connecting the Geiger signal to one of the input pins in the parallel port (see figure 1(a)).

For the standard parallel port, pin 11 is an inverting input and so will be most convenient if the output signals from the Geiger are negative logic (logical_1 = 0 V/logical_0 = 5 V). Any of the other input pins should be used if Geiger signals are positive logic. In our particular case the output pulses from the Geiger electronics were very long, and that caused our computer to count each pulse several times. Shortening of the pulses was accomplished with a simple RC circuit (figure 1(b)) which uses a +5 V output from pin 9 (OUT P,128 when initiating the program).

The required software is also extremely simple. For instance, in BASIC, just the following three lines suffice:

```
10 DEF SEG=0:P=PEEK(1032)+256*PEEK(1033) 'Localize port direction
20 WAIT P+1,128:PRINT TIMER 'Wait for signal, annotate arriving time
30 IF INKEY$="z" THEN STOP ELSE GOTO 20 'Repeat loop and stop control
```

In the above example, the 128 value in the WAIT command assumes the 11 input pin is being used. Respectively values 64, 32, 16, 8 would be required if input pins 10, 12, 13 or 15 were used. Additional extensions of the above programs could be as follows.

(1) Inserting a software dead time for didactic purposes (see section 4). This can be easily achieved by inserting a delay loop inbetween lines 20 and 30.
(2) Printing to a file. This can be easily accomplished by initiating OPEN 'o',#1, 'filename' and substituting 'PRINT TIMER' by 'PRINT#1 TIMER'.
(3) The time origin can be discounted by a first sentence T0 = TIMER, and substituting 'PRINT TIMER' by 'PRINT TIMER-T0'.
(4) Instead of the 'z-push' stop procedure, the data acquisition process can be interrupted after suitable predetermined conditions with simple modifications of the code. For instance after a given number of recorded events (by inserting an event counter or FOR-NEXT loop), after a time interval TMAX (inserting an IF (TIMER-T0) > TMAX THEN STOP) or at a fixed time TFIX (inserting an IF TIMER > TFIX THEN STOP).

This technique has been applied in our student lab using various PCs ranging from Pentium I to Pentium IV. The BASIC control program has been compiled and the executable file run on any Windows operating system. Under MS-DOS and Windows 3.x/95/98/ME operating systems, applications can directly access system hardware and consequently, in any of those environments the very simple procedure described here will work correctly. Being more secure environments, Windows NT/2000/XP operating systems assign some privileges and restrictions to different types of programs, and direct port access is forbidden for user applications. Consequently, under these operating systems, previous installation of some 'kernel mode drive' or 'privilege remapping' may be necessary in order to assure parallel port access. Simple tools for that purpose can be found at many Internet sites [7] and are also well described in the technical literature [8].

The set-up consists of a Geiger counter, a low activity radioactive source (e.g. 2 μC of ^{60}Co) and the PC. The Geiger output signal is directed to the PC parallel port as explained above. As a result of data acquisition an ASCII file is written which consists of a certain number, N, of values, $t_1, t_2, \ldots, t_i, \ldots, t_N$, each being the time when the Geiger was fired. As shown in next few sections, this time series allows both counting and timing of the particles.

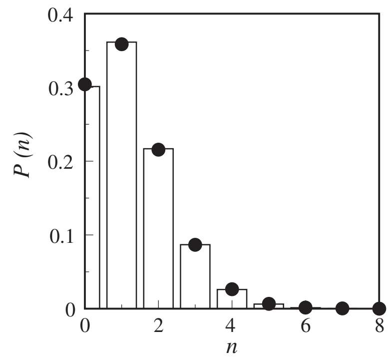

Figure 2. Experimental probability distribution of n events in a counting period of 0.5 s (filled circles) and the corresponding Poisson distribution for $\langle n \rangle = 1.20$ (bars).

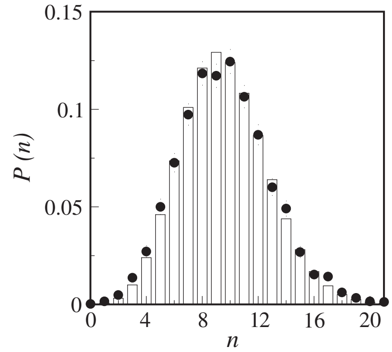

Figure 3. Experimental probability distribution of n events in a counting period of 4.0 s (filled circles) and the corresponding Poisson distribution for $\langle n \rangle = 9.60$ (bars).

3. Checking the Poisson statistics

Let us assume a radioactive source emitting particles which are detected by a counter at a rate A. The number of particles n detected in a time Δt is a statistical variable following the Poisson law

$$P(n) = \frac{\langle n \rangle^n}{n!} \, \mathrm{e}^{-\langle n \rangle} \tag{1}$$

with average value $\langle n \rangle = A \times \Delta t$.

The data recorded by our simple set-up allows us to check equation (1). First a time period Δt is chosen. Then the series is split into $M = T/\Delta t$ intervals, where the total data-taking time T should be as large as possible. For $m = 1, 2, \ldots, M$ the number of events t_i satisfying $(m-1)\Delta t < t_i < m\Delta t$ is counted. Following this procedure a histogram of the event number with M entries is achieved. Since Δt can be freely chosen after data taking, one time series allows checking equation (1) for different $\langle n \rangle$ values.

As an example, the results of an acquisition period of $T = 13\,100$ s with a total of $N = 31\,439$ events (average rate of about 2.4 Hz) have been studied. For this measurement no artificial dead time has been applied in the writing algorithm (see sections 2 and 4). Figures 2 and 3 show $P(n)$ versus n for Δt values of 0.5 and 4.0 s respectively. Data (filled circles) are

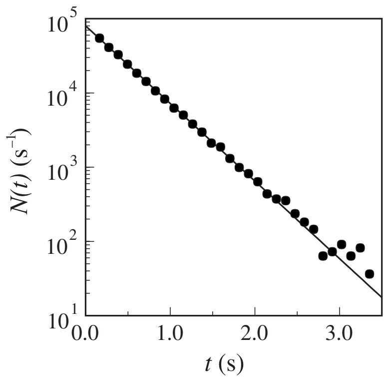

Figure 4. Histogram of time intervals for a counting rate of $A = 2.40\ \mathrm{s}^{-1}$ (circles). The continuous line represents a fit to equation (3).

compared with the Poisson statistics (bars) for the experimental mean value $\langle n \rangle = N\Delta t/T$. In these tests $\langle n \rangle$ equalled 1.20 and 9.60 for Δt values of 0.5 and 4.0 s respectively. Figures 2 and 3 show a very good agreement between theory and experiment. As is well known the standard deviation of the Poisson distribution is $\sigma(n) = \sqrt{\langle n \rangle}$ [3]. In our example the theoretical value of $\sigma(n) = \frac{\sqrt{N}}{T}\Delta t$ was 1.095 and 3.098, in agreement with the standard deviation of the experimental distributions, 1.10 and 3.25 for counting periods of 0.5 and 4.0 s respectively.

The probability distribution of time intervals separating random events is of practical interest in radiation measurements [3]. The probability for a detection to be followed by the next one in time between t and $t + \mathrm{d}t$ is given by

$$P(t)\,\mathrm{d}t = A\,\mathrm{e}^{-At}\,\mathrm{d}t. \tag{2}$$

This result can be easily deduced from equation (1) since $P(t)\,\mathrm{d}t$ is the probability of zero detections in a time interval t and one detection in between t and $t + \mathrm{d}t$. $P(t)$ can be measured from our time series by representing on a histogram the time difference between successive events, $t_{i-1} - t_i$. This histogram has been represented in figure 4 which shows a linear relation between the logarithm of the probability and t. The corresponding normalized distribution fits an exponential function

$$P(t) = A_1\,\mathrm{e}^{-A_2 t} \tag{3}$$

with parameters $A_1 = 2.41 \pm 0.02\ \mathrm{s}^{-1}$ and $A_2 = 2.40 \pm 0.07\ \mathrm{s}^{-1}$. The expected result from equation (2) is $A_1 = A_2 = A = 2.400 \pm 0.013\ \mathrm{s}^{-1}$ since $A = N/T$ with statistical uncertainty of $\sqrt{N}/T$.

Note that in equation (2) the origin can be any random time not necessarily related to a previous counter signal. This well-known fact can be easily checked using our data file. In the first place, a sample of random times t_r uniformly distributed between t_1 and t_N is generated. Then a histogram of $t_r - t_j$ where $t_{j+1} < t_r < t_j$ provides the probability distribution of time intervals with origin unrelated with counter signals.

In general the probability of detecting a group of m events after a previous one, that is the probability of $m - 1$ events in the time interval 0–t and one event between t and $t + \mathrm{d}t$ is given by

$$P(t)\,\mathrm{d}t = \frac{A^m t^{m-1}}{(m-1)!}\,\mathrm{e}^{-At}\,\mathrm{d}t \tag{4}$$

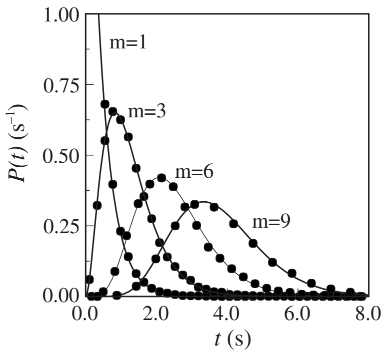

Figure 5. Probability time distribution for groups of $m = 1$, 3, 6 and 9 events. The experimental data (filled circles) agree with the theoretical expectations (continuous line) of equation (4).

where A is the average rate of events [3]. This situation is found when the detector signals are sent to a data buffer which produces an output pulse when m input signals have been accumulated allowing data recording at a lower rate. Obviously equation (2) is a particular case ($m = 1$) of this general distribution. Our data file allows an experimental verification of equation (4). In figure 5 the normalized histogram of $t_{i-1} - t_{i-1+m}$ for groups of $m = 1$, 3, 6 and 9 events has been represented in very good agreement with the theoretical expectations (the continuous line). The origin of each group t_{i-1} has been chosen equal to the arrival time of the first event after the previous group and thus every event has been only included in one group.

The standard deviation $\sigma(t)$ for the probability distribution of m events fulfils the following relation [3]

$$\frac{\sigma(t)}{\langle t\rangle} = \frac{1}{\sqrt{m}} \tag{5}$$

indicating that the per cent fluctuations in the time interval for a group of events decrease with the group size. In the limiting case $m \to \infty$, an output signal in coincidence with the end of the group behaves as good as a clock, at a rate of

$$\frac{1}{\langle t\rangle} = \frac{A}{m} \tag{6}$$

with no fluctuations. As a further exercise, we suggest readers test experimentally equation (5) with our technique by recording a large data sample (e.g. 24 h data taking at a rate of 3 Hz).

4. Dead-time effects on the Poisson statistics

In general, all particle counters become 'blind' for a certain time after a detection. The origin of this dead time depends on the counter design. For instance in Geiger tubes, an ionizing particle produces a discharge. Until the discharge is fully quenched, a further particle does not produce any observable effect on the detector. In addition, very often data acquisition contributes to the total dead time, because while a signal is being processed no further event is allowed in the counting chain. The artificial dead time introduced in our writing algorithm (see section 2) emulates this acquisition dead time.

Results of the Poisson statistics rely on the fact that the events are random, that is there is no correlation among them. Obviously dead time introduces a constraint in the time interval

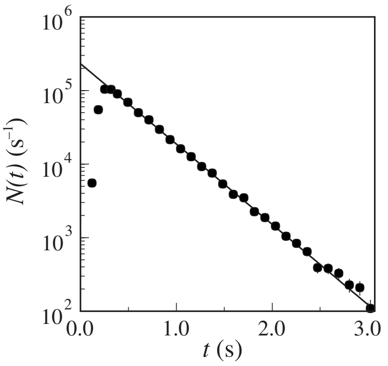

Figure 6. Histogram of time intervals for a counting device with a dead time of $\tau \approx 0.2$ s (filled circles). Note the sharp cut at $t = \tau$. The straight line is a fit to equation (3) for $t > \tau$ which gives a value of the true rate $A_v = 2.48$ s^{-1}.

between successive events. Thus, both distributions $P(n)$ and $P(t)$ are expected to deviate from equations (1) and (2) respectively when using detectors and/or acquisition systems with non-negligible dead times.

Using our device, a time series has been acquired with an artificial dead time τ of about 0.2 s larger than the intrinsic Geiger dead time. The sample consisted of $N = 52\,936$ events in an acquisition time of $T = 31\,510$ s. As expected, $P(t)$ shows a sharp drop below $t = \tau$ and $P(t < \tau) = 0$ (see figure 6). For $t > \tau$ the distribution is not modified and thus it should follow equation (2). Experimental data fit equation (3) with $A_1 = 2.52 \pm 0.03$ s^{-1} and $A_2 = 2.51 \pm 0.01$ s^{-1}. From this result a true rate, A_v, of 2.48 s^{-1} is inferred. The measured rate $A_m = N/T = 1.680$ s^{-1} is significantly lower because of dead-time losses. The relation between A_m and A_v is given by the simple formula[1] [3]

$$A_v = A_m/(1 - \tau A_m). \tag{7}$$

Dead time can be obtained from equation (7) since both A_m and A_v are accurately measured. In our case a value of $\tau = 198 \pm 4$ ms is inferred, consistent with that obtained from the sharp cut in figure 6. This technique, which provides an accurate measure of dead-time loses [6], is often used in real experiments [9].

The effect of dead time on the $P(n)$ distribution can also be studied with our set-up. The corresponding theory has been developed in detail by Müller [10]. According to equation (7) the modified distribution $P_m(n)$ has a mean value

$$\langle n \rangle_m = \frac{\langle n \rangle}{1 + \tau A_v} \tag{8}$$

where $\langle n \rangle = A_v \Delta t$. The standard deviation of the distribution $\sigma_m(n)$ is shortened by the law [10]

$$\sigma_m^2(n) = \frac{\sigma^2(n)}{(1 + \tau A_v)^3} = \frac{\langle n \rangle}{(1 + \tau A_v)^3}. \tag{9}$$

The above equation is valid as long as the resulting $\sigma_m(n)$ is not too small ($\sigma_m(n) \geqslant 1$ is enough). Therefore the effect of dead time is a shifting and narrowing of the original Poisson distribution.

[1] In fact this relation is only valid for the so-called *non-paralysable* model of dead time. In counters following this model, like our case, dead time is not extended by particles arriving while the detector is off due to dead time from a previous particle.

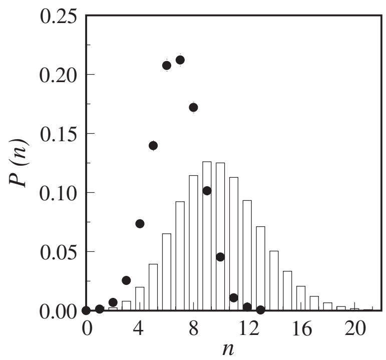

Figure 7. Probability distribution of n events with a dead time of about 0.2 s in a counting period of 4.0 s. The measured distribution $P_m(n)$ (filled circles) is significantly shifted and narrowed as compared with the Poisson $P(n)$ distribution (bars).

The above data series registered with dead time has been used to experimentally study these features. An interval $\Delta t = 4$ s has been chosen. The experimental distribution $P_m(n)$ (filled circles) has been compared in figure 7 with the Poisson distribution $P(n)$ (bars) for $\langle n \rangle = 9.92$ (this value has been inferred from the true rate given by the time interval distribution). The figure clearly shows the shifting and narrowing of the distribution predicted above.

Dead time can be determined by the simultaneous measurement of mean value and width of $P_m(n)$. From equations (8) and (9)

$$\tau A_v = \sqrt[3]{\frac{\langle n \rangle_m}{\sigma_m^2}} - 1 \tag{10}$$

is inferred. The standard deviation of the measured $P_m(n)$ distribution is found to be $\sigma_m(n) = 1.79$ while $\langle n \rangle_m = 6.72$. From these values a dead time of 190 ms is inferred (equation (10)), in good agreement with the result obtained from the time interval distribution.

5. Conclusions and further suggestions

A simple procedure has been shown to write onto a file the time at which a Geiger counter is fired. The counter output feeds the parallel port of a PC without any need for further electronic cards or modules. By means of this device and a low activity radioactive source, a large data sample can be stored allowing accurate experimental studies of the properties of counting statistics. In the first place both the distribution probability $P(n)$ of counting n events in a given time and the probability distribution of time intervals $P(t)$ are found to follow very precisely the predictions of the Poisson statistics. In addition, properties of the probability distribution of groups of events can be studied.

A delay loop has been introduced in the acquisition software which enlarges the intrinsic Geiger dead time. The effect of dead time on both $P(t)$ and $P(n)$ can be studied in detail. From both distributions an accurate measure of the true rate can be performed.

From the file written by the PC many other interesting properties of the counting statistics can be shown in the student laboratory. We suggest a further exercise related to the effect of dead time on the $P(t)$ distribution. In the case of dead time, the probability distribution of time intervals with random origin (see section 3) is demonstrated to follow the law [10]

$$P(t) = A_v \exp(-A_v \tau) \qquad \text{for} \qquad t < \tau \tag{11}$$

$$P(t) = A_v \exp(-A_v t) \qquad \text{for} \qquad t \geqslant \tau. \tag{12}$$

This result can also be checked using our technique by recording a series similar to that in section 4 and applying the procedure described in section 3 for a random origin.

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