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Letters and comments

Experimental statistics for undergraduates—revisited

1. Introduction

The work described here is based on a series of articles (MacLeod *et al* 1976, MacLeod 1976, 1980) in which a laboratory radioactive source was used as a source of random events, which enabled an investigation of the Poisson distribution to be made, and was of considerable benefit in giving undergraduate students an understanding of counting statistics. The second article in this series (MacLeod 1976) gives details of the circuitry used for counting, while the third (MacLeod 1980) describes the circuitry used to obtain a graphic display of the data on a television screen. The second article is an interesting and informative exercise in digital electronics, and the third article is useful in understanding video display circuitry. However, if one is interested only in the statistics of the experiment, both of these exercises are unnecessary.

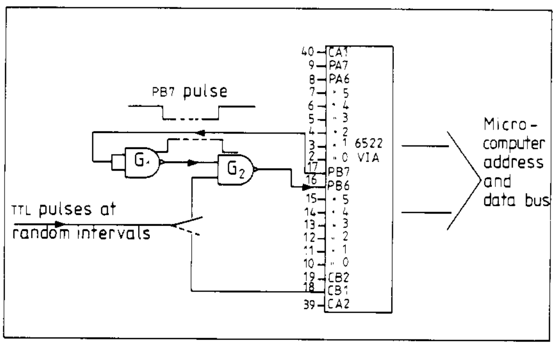
The present work shows that the counting circuitry (34 of the 74 series integrated circuits (MacLeod 1976)) may be replaced by a pair of NAND gates and a 6522 versatile interface adapter (VIA). The display circuitry is already available in almost every popular make of microcomputer on the market today. Furthermore, many of these micros have a 6522 VIA available on board for the user as an interface device, so that the only circuitry external to the microcomputer for the purpose of counting is a pair of NAND gates. MacLeod (1980) considers the possibility of using a microcomputer to perform the functions outlined in his article, but dismisses the idea on the basis that no significant reduction in specialised hardware would result. This work shows that with the aid of a 6522 VIA, a very significant reduction in specialised hardware is obtained, while the features of the dynamic display, outlined by MacLeod, are still retained.

2. Apparatus

A radioactive source, amplifier and pulse squarer similar to those described by MacLeod *et al* (1976) are used to provide a continuous stream of TTL pulses occurring at random intervals. One then needs to count repeatedly the number of pulses occurring in a given fixed period, with some method of recording each result. Finally a histogram plot of the frequency of a given count against count number, together with a method of calculating the mean, variance and χ^2 fit are required.

The central element in this work is the 6522 VIA, which is a sophisticated integrated circuit used as a peripheral interface to many microprocessors. It has 16 internal registers, but we will be concerned with only three of these, labelled as T_1, T_2 and CONTROL register. T_1 and T_2 are both 16 bit registers and they may be used as either timers or counters depending on the value in the CONTROL register. T_1 is used here to provide a fixed time interval, while T_2 is used to count all the pulses that occur during this interval. When T_1 is used as a timer in the one-shot mode (cf SY6522 data sheet, Synertek Inc.), an output pulse appears on pin PB7 of the 6522, the length of which is proportional to the value originally loaded into the T_1 timer. This pulse is then used to gate the random pulses via the two NAND gates, from the radioactive source to pin PB6 of the 6522. When T_2 is initialised to act as a counter, all pulses fed to pin PB6 are recorded up to a maximum of the clocking rate of the 6522 (1 MHz). Figure 1 shows the simplicity of the interface to the 6522, where gate G_1 is used to invert the output pulse on PB7. With a clocking rate of 1 MHz the length of the pulse from the T_1 timer may take any value in the range 1–65 535 μs; typically times in the range 1–65 ms are used as a suitable counting period. The results in this article were obtained using a 6522 onboard a BBC microcomputer and the data were plotted using the graphics available. After each counting period, the value of the T_2 counter is read and the counter is then restarted. During the next counting period, the data from the previous sample are

Figure 1 Interface of random pulses to microcomputer. The random pulses are either gated through G_2 to PB6 for the Poisson distribution or fed directly to CB1 for the exponential curve. (In the present work the 6522 VIA is onboard a BBC microcomputer and all the required connections are available through the user port).



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accumulated and the display is updated; this requires a maximum of 0.9 ms when performed in machine code. Thus, using a sampling period greater than 1 ms allows the counting and plotting to be performed simultaneously, and almost all the integration period is spent in the counting mode. When the required number of samples has been obtained the processor is dedicated to calculating the mean, variance and χ^2 fit for the data.

The advantages of the system are that the apparatus for the statistics experiment may be completely assembled in a matter of minutes, rather than going through MacLeod's more complicated procedure. The counting interval, the total number of samples and the histogram scaling may all be set in software; software allows far more flexibility than can ever by achieved with a set of switches. The counting rate is 10 times faster than in the earlier work, allowing data to be accumulated more quickly. Using a suíte of programs, the student can be taken step by step through a learning sequence, starting with a single-channel counter and plotting the results by hand, up to a full multichannel system with theoretical curves calculated and displayed along with experimental data for visual comparison. If a high level language, e.g. BASIC, is used for processing and plotting the results the student may readily change the program and may well learn even more from the experiment. A print-out of the results may be obtained, which allows comparison of many sets of results, and serves as a permanent record for the student's laboratory notebook.

2.1. Exponential curve

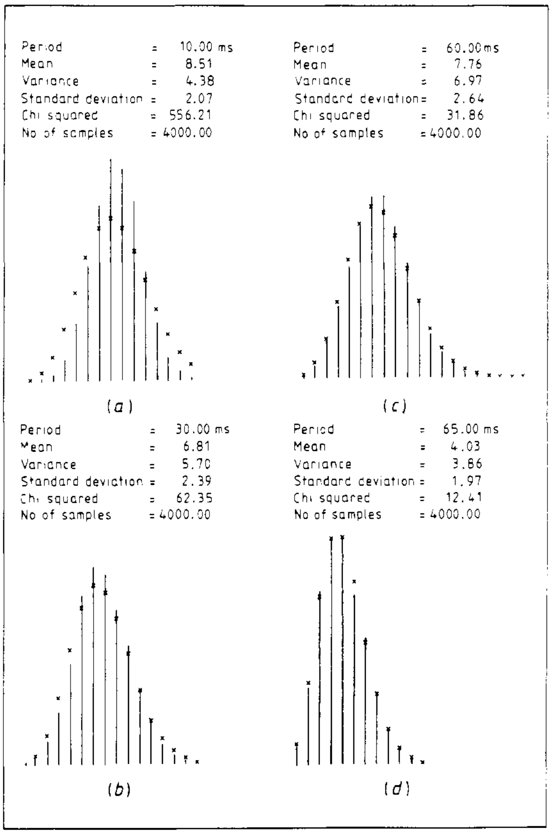
MacLeod (1980) shows how his circuitry was used to demonstrate the exponential curve, by measuring the time between consecutive pulses and plotting a histogram of the frequency of occurrence of a given time against time. This useful demonstration may also be obtained with the present system by applying the TTL random pulses to pin CB1 of the 6522 and using T_1 as a timer to measure the time between pulses. A type of direct memory access per 500 events, to ensure minimum distortion to the exponential curve, is used by MacLeod; the simplicity of the interface used here would be lost if such a scheme were attempted, and thus an interrupt system is used to register the arrival of each event. This adds a small delay (7 μs) to each time and may cause some distortion of the exponential for pulses which occur very close together. However, because of the ten-fold increase in clock speed, together with relatively low count rates (easily adjusted by increasing the distance between source and detector), quite acceptable results have been achieved as shown in figure 3.

3. Software

The program to access the 6522 VIA and plot the histogram of the results was originally written in BASIC. Each point in the display required 20 ms to be accumulated and plotted, with the result that an integration of 10 000 samples, each of 1 ms duration, required 10 s for the actual counting and 190 s for plotting and processing. Nevertheless a 10 000 sample integration could be obtained in under 4 min, which is a very short time in terms of a student laboratory practical session. Writing the programme in BASIC has the added advantage that the lower-level student may readily make alterations to the program.

The program was rewritten in 6502 code and each point then required only 0.9 ms to process and plot. Choosing a sampling period of 1 ms or greater allows the integration and plotting to occur simultaneously, and almost 100% of the integration time is spent in actual counting. Since the random pulses are gated into the T_2 counter by the T_1 pulse, sampling periods as short as 1 μs may be chosen, but again much of the integration time is spent in processing and plotting rather than in counting. The software to calculate the mean, variance and χ^2 fit is

Figure 2 Poisson distributions redrawn from print-out. Bars show experimental values. Crosses show theoretical values for a spectrum with the same mean value. The four figures were obtained with the following count rates: (*a*) 860 s^{-1}, (*b*) 240 s^{-1}, (*c*) 130 s^{-1}, (*d*) 60 s^{-1}.



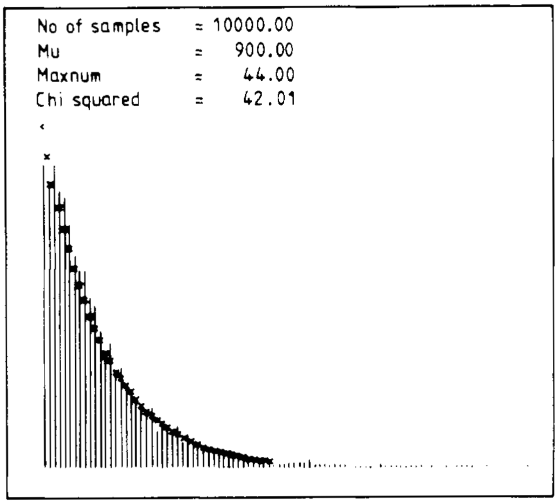


Figure 3 Exponential distribution (time between pulses) redrawn from print-out. Bars show experimental values and crosses show best-fit exponential. Maxnum is the maximum x coordinate used in the χ^2 fit. Mu is the calculated number of counts per second based on the fit.

still written in BASIC, as is the χ^2 fit for the exponential curve. It is, however, essential to use 6502 code to access the 6522 in the case of the exponential curve, in order to minimise the delay between the arrival of a pulse and the T_1 timer being read.

4. Results

The results in this work were obtained using a Sr(90) 5 μCi β source and a simple laboratory Geiger counter. As MacLeod (1976) has pointed out, the Geiger counter pulses are very long ($\sim 200\ \mu$s), which means that the pulses do not occur at truly random times. However, the effect of the long dead time is reduced if low count rates are used. Figures 2(*a*)–(*d*) show the improvement in χ^2 fit as the count rate was decreased from 800 s^{-1} to 60 s^{-1}. The figures show that quite reasonable results may be obtained at these count rates and that the experiment is readily available to almost all physics laboratories, without the need for specialised hardware. Figure 3 shows the results of feeding the random pulses to pin CB1 of the 6522 and counting the time between pulses; again the results compare favourably with the earlier work.

5. Conclusion

The use of microcomputers for monitoring experiments and collecting data is a commonplace occurrence in physics laboratories today. In this work it is the power and elegance of the 6522 interface chip which allows the apparatus to be simplified so much. The critical part of actually performing the counting is performed by the hardware of the chip, while the less critical parts of display are performed in software using the already available video circuitry.

References

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The entropy of a damped harmonic oscillator

This letter arises out of the Ginsburg–Landau theory of critical points (Shang-Keng Ma 1976), in which fluctuations of the order parameter are Fourier-analysed, and each term (in a quadratic approximation) is treated as contributing to the partition function as if it were a harmonic oscillator. Usually, however, the energy attributed to the fluctuation is purely potential, i.e. proportional to η_k^2, where η_k is the amplitude of a component with wavenumber k, and with no term in $\dot{\eta}_k^2$. To use an electrical analogy which will prove helpful later, the system is more like an RC circuit than a resonant LC circuit; or like a very highly overdamped harmonic oscillator. What is the entropy of such a system? On the answer to this question depends the contribution of fluctuations to the specific heat anomaly. For so simple a question the discussions in the literature are surprisingly complicated, and the treatment given below is an improvement, if only because the means are commensurate with the task. An account of earlier work may be recovered from the references cited by Lonzarich (1986).

To derive an expression for the entropy (which turns out to involve fewer tedious details relating to convergence of integrals than does the free energy), we must include the damping mechanism explicitly – we need to find how the entropy of a complete system is changed when it is linearly coupled to an oscillator in such a way that the oscillator is exponentially damped. The circuit of figure 1 is such a system, being a simplified version of an arrangement analysed elsewhere (Pippard 1983). An infinitely long loss-free transmission line, of characteristic impedance Z_0, behaves like a pure resistance Z_0 in the circuit, and by varying Z_0 the circuit may be lightly damped or